Considerations for Acceleration of Deuterons and He-3 in RHIC

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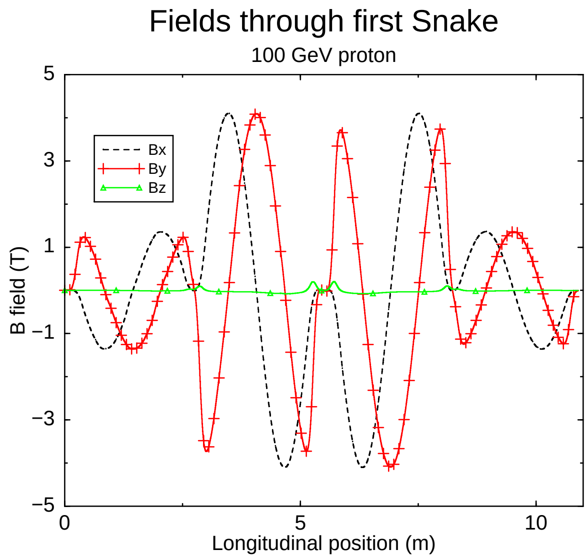
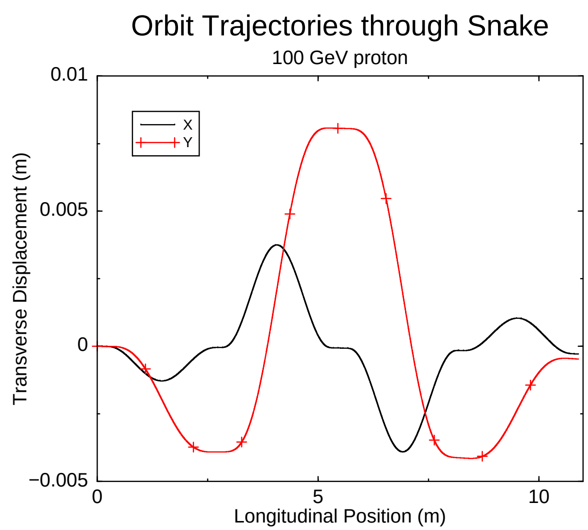
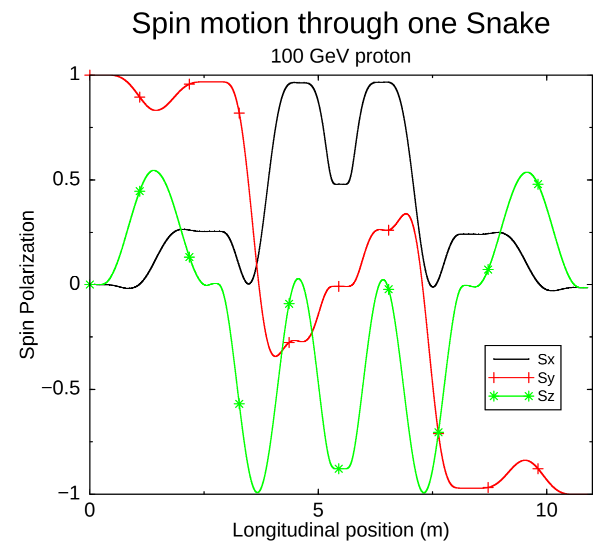
RHIC Spin Params for Diff. Species

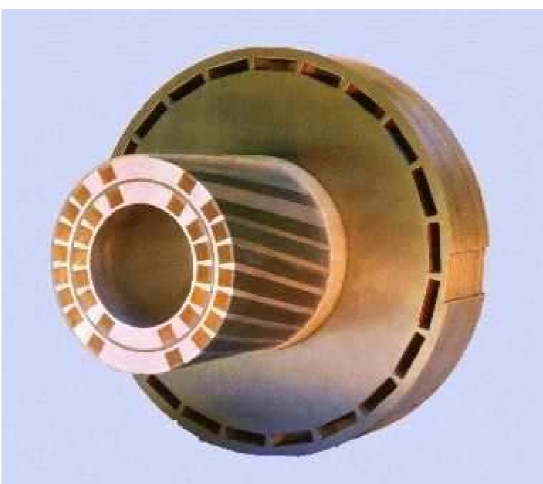
	p	${}^2_1\mathrm{H}^+$	${}^3_1\mathrm{H}^+$	${}^3_2\mathrm{He}^{+2}$
M [GeV/c^2]	0.938272	1.875613	2.808921	2.808391
μ/μ_N	2.792847	0.857438	2.972962	-2.127498
$G = (g-2)/2$	1.792847	-0.142987	7.918171	-4.183963
mc^2/G [MeV]	523.3	-131117	354.7	-671.2
$(p/q)_\mathrm{inj}$ [Tm]	79.367	80.704	57.819	55.216
U_inj [GeV]	23.812	24.267	17.560	33.226
U_inj/n [GeV]	23.812	12.134	5.853	11.075
γ_inj	23.379	12.938	6.251	11.831
$G\gamma_\mathrm{inj}$	45.500	-1.850	49.500	-49.500
$(p/q)_\mathrm{max}$ [Tm]	833.904	833.904	833.904	833.904
U_max [GeV]	250.000	250.005	250.014	500.004
U_max/u [GeV]	250.000	125.003	83.338	166.668
γ_max	266.447	133.293	89.007	178.039
$G\gamma_\mathrm{max}$	477.699	-19.059	704.773	-744.910

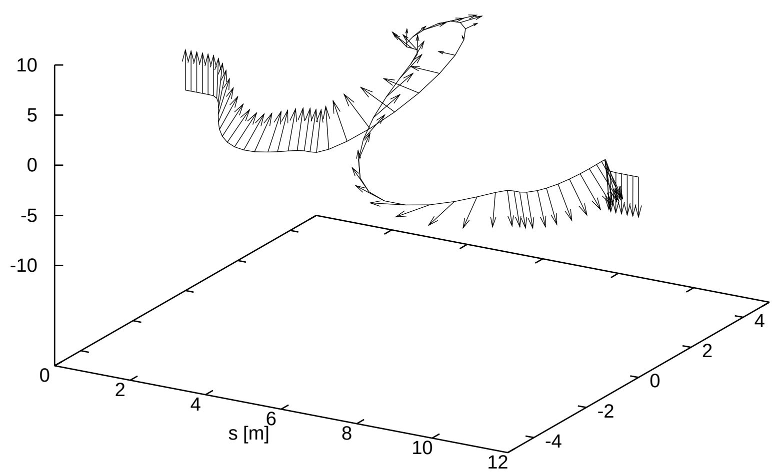
Trajectory and Spin through Snakes











2d plots by V. Ranjbar

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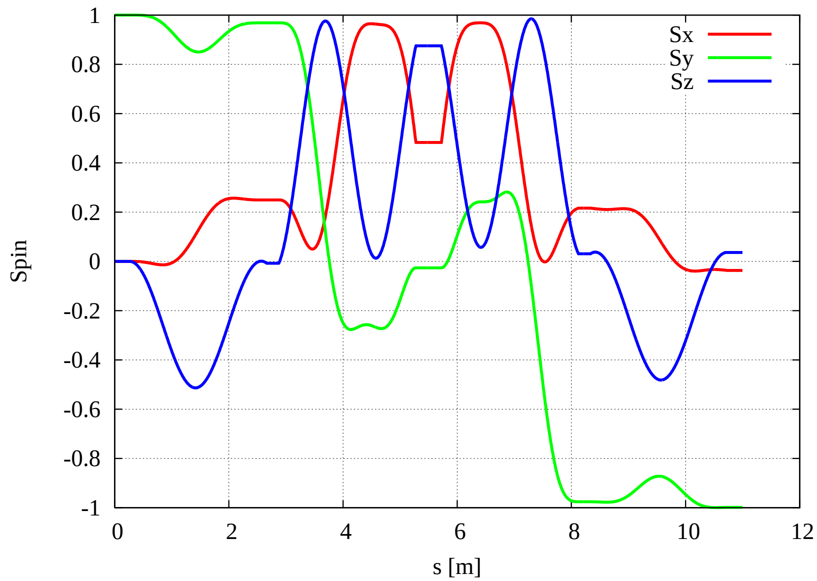
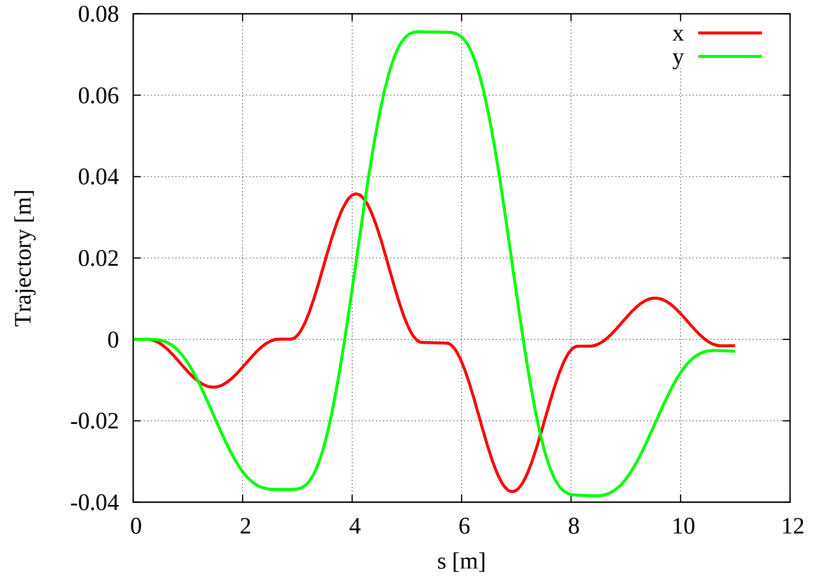
Machine Advisory Committee Review
Waldo MacKay 15 September, 2010

Scaling RHIC Snake for Deuterons





$B_{\mathrm{out}} = 33.5$ T $\quad B_{\mathrm{in}} = 101.6$T

Trajectory for 250 GeV $^2H^+$.

$$B_{\mathrm{d}} = \frac{G_{\mathrm{p}}\gamma_{\mathrm{p}}}{G_{\mathrm{d}}\gamma_{\mathrm{d}}} B_{\mathrm{p}} = \frac{477.7}{-19.06} B_{\mathrm{p}}$$

Even if we could build these magnets, we couldn't inject.

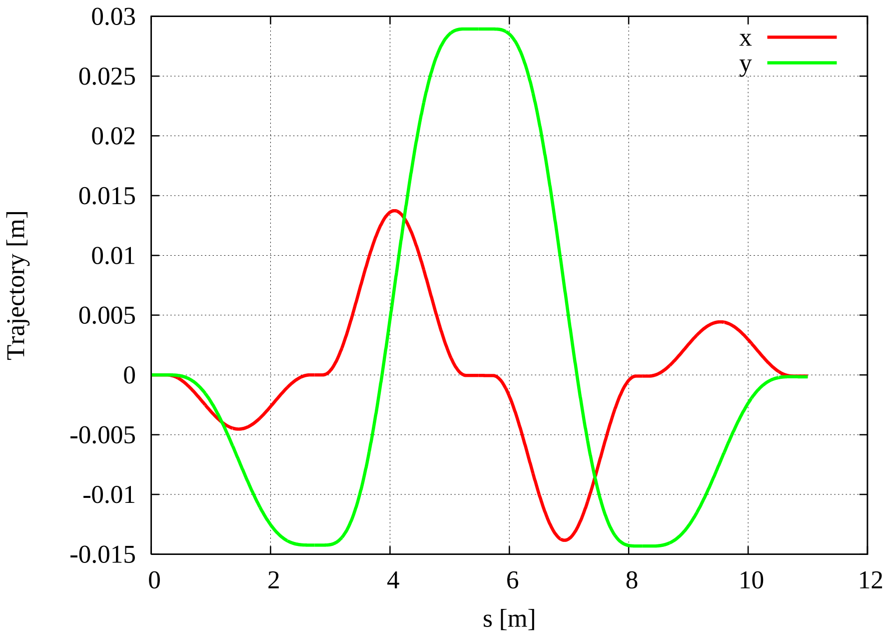
Problem with deuterons in RHIC

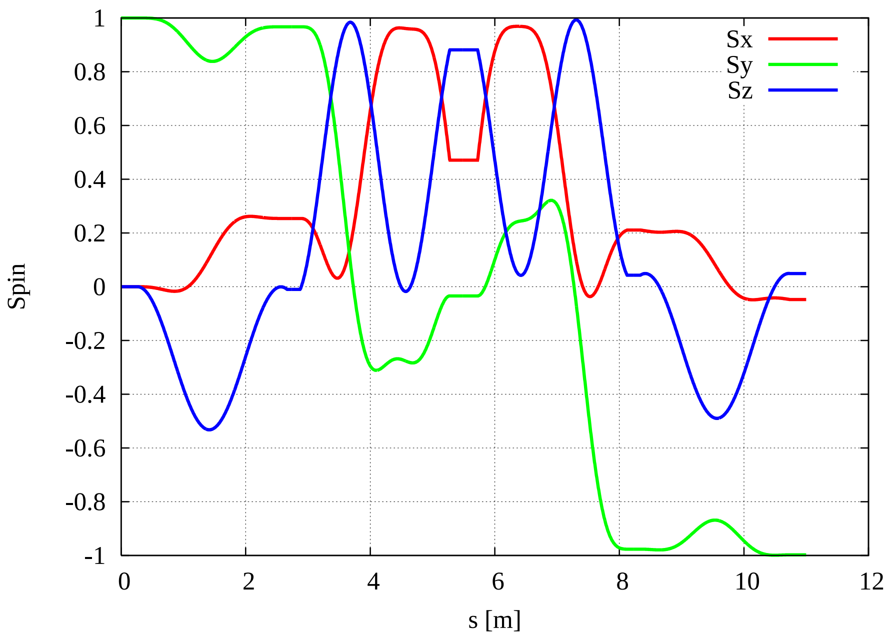
- G is too close to zero for deuterons.
- Snakes not strong enough to do anything.
  - At top energy 250 GeV helix precession angles $\propto B^2$.
    - Strength $\sim \frac{4\ \mathrm{T}}{100\ \mathrm{T}} \sim 0.16\%$.
    - Tracking gives a snake strength of 0.06%.

- For more $$$ perhaps we might get be able to build a 30 m long large aperture partial snake with a strength of a few percent?
- AC dipole to flip at strong intrinsic resonances probably not practical.
- Spin rotators have the same scaling problem as snakes.

- Bottom line: **Polarized deuterons in RHIC will be very hard.**

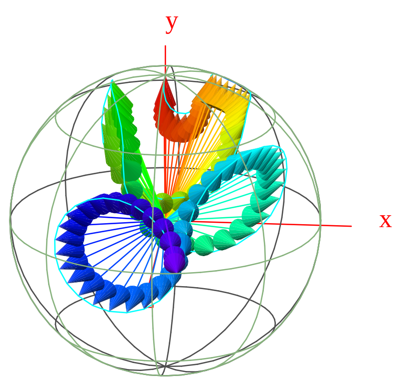
RHIC snakes with He-3 at injection



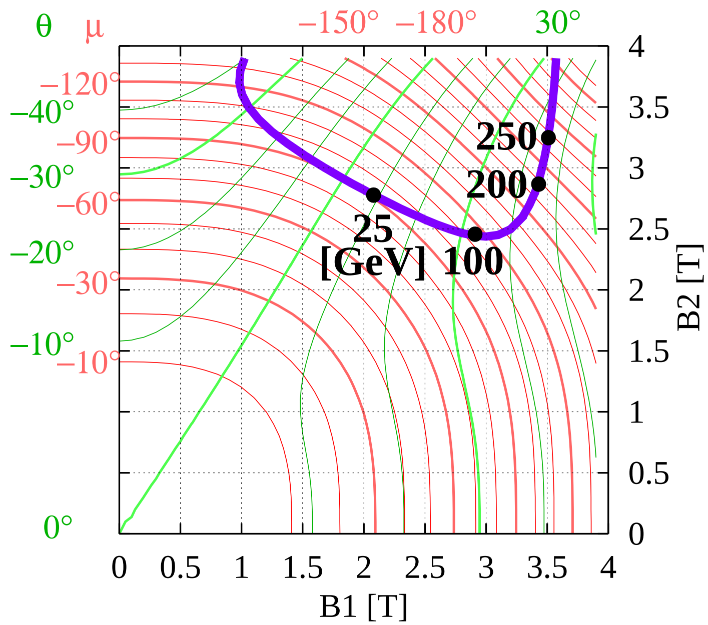


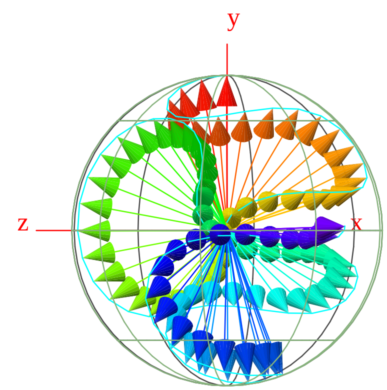
- Injection: $U = 33.226$ GeV
- Aperture looks fine: $\Delta y < 3$cm.

Rotators and D0-DX Bends for protons



Rotator's spin vector at injection energy

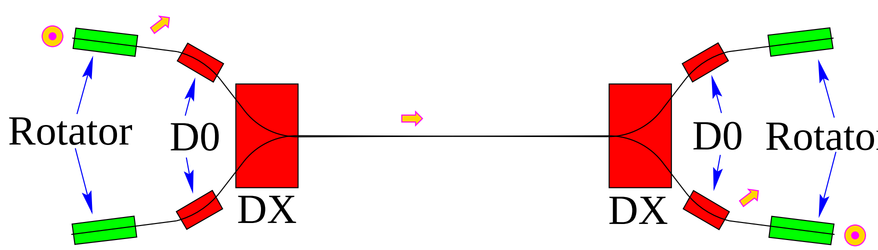




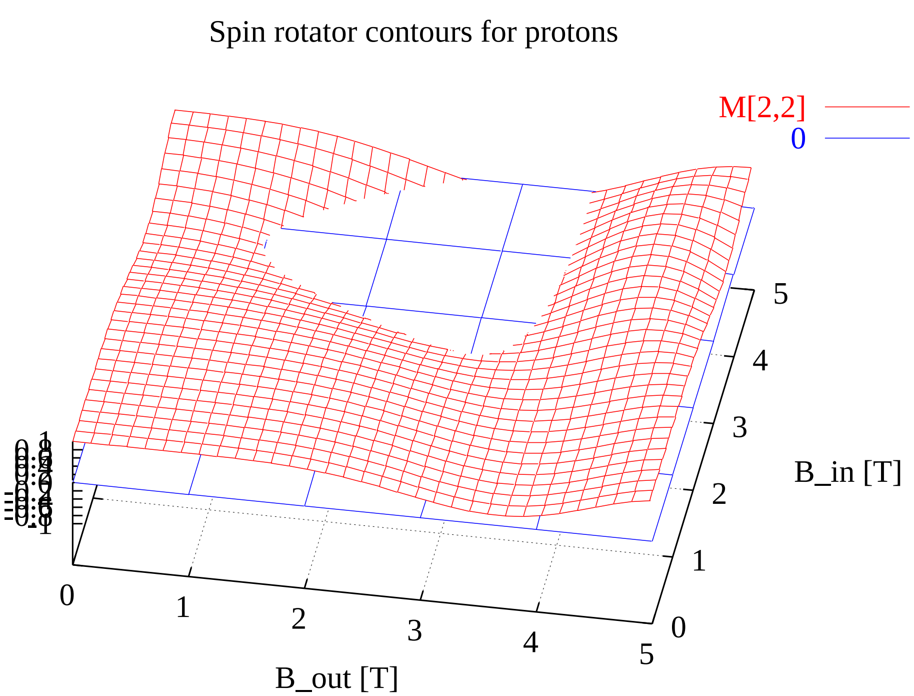
Rotator's spin vector at 250 GeV

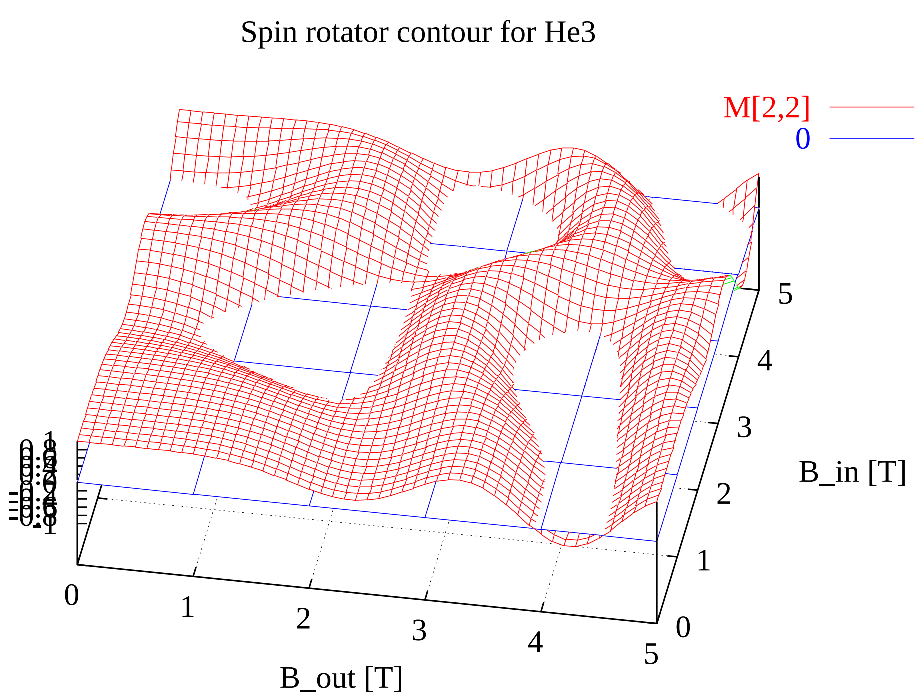
$E = 25$ GeV
D0DX: 10° precession

$E = 250$ GeV
D0DX: 100° precession



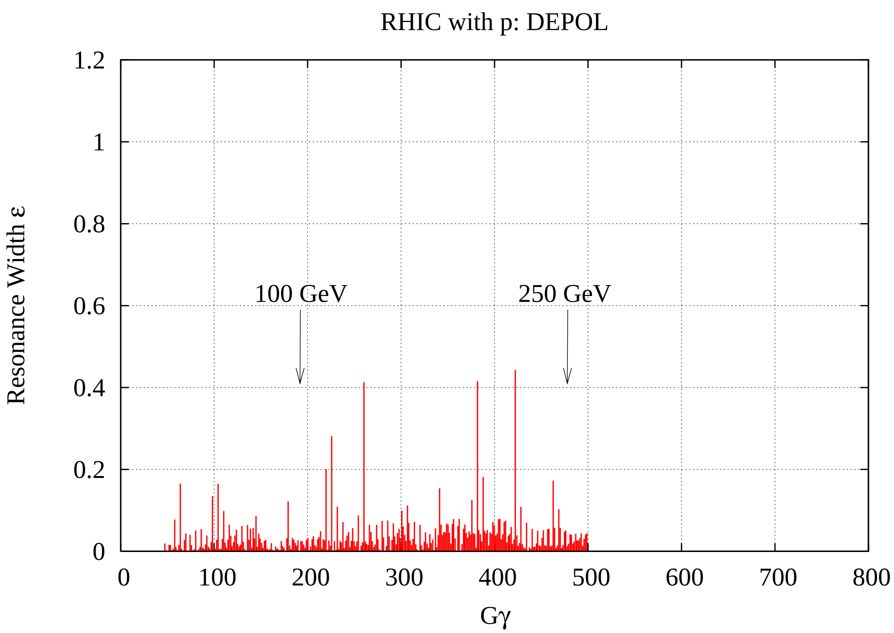
Comparison of Rotators for ^{3}He and p

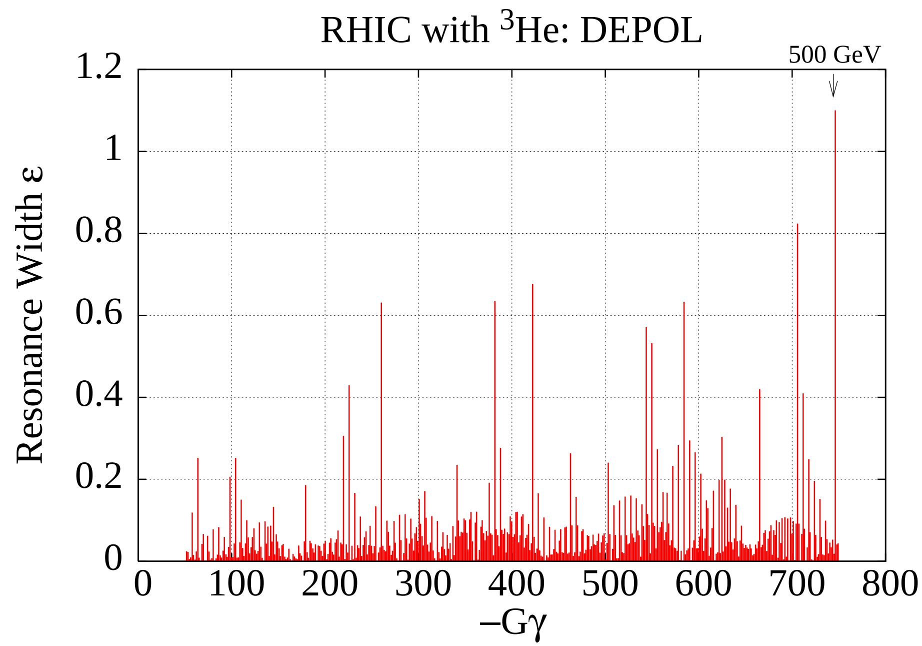




- Rotators easier to rotate vertical spin into any direction in horizontal plane.

Intrinsic Resonances in RHIC

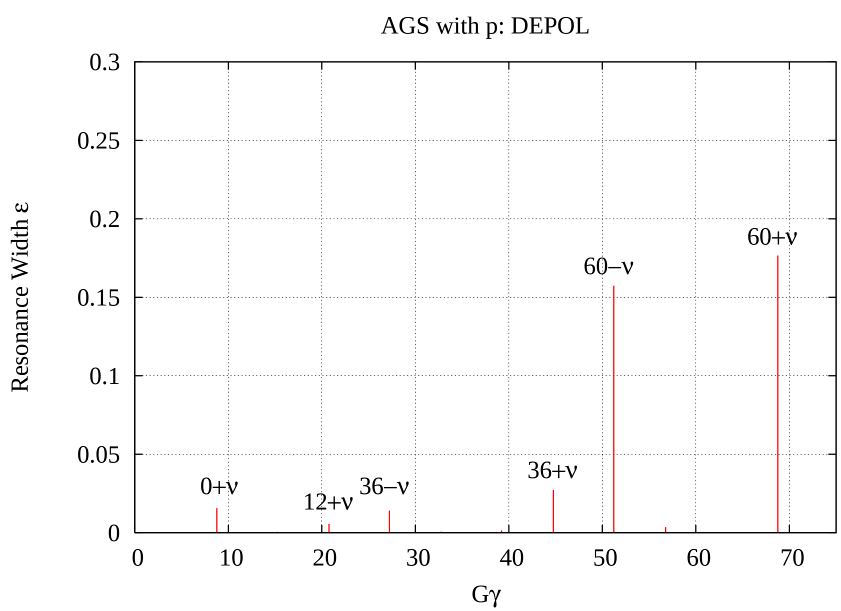


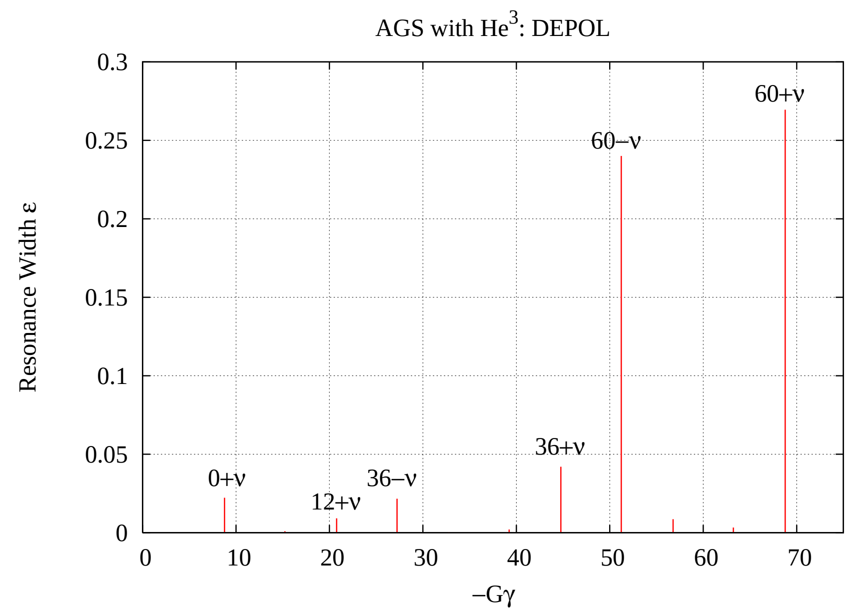


$$|G\gamma|_{\max} = \begin{cases} 478, & \text{p} \quad (250\text{ GeV}) \\ 745, & {}^3\text{He} \quad (500\text{ GeV}) \end{cases}$$

Note: DEPOL calculations do not include snakes and rotators.

Intrinsic Resonances in AGS





$$|G\gamma|_{\rm ext} = \begin{cases} 45.5, & \text{p} \\ 49.5, & {}^3\text{He} \end{cases}$$

Extract below the $\nu_{\rm sp} = 60 - Q_{\rm v} \sim 51$ resonance.
AGS has a 12-fold superperiodicity

He3 Spin Resonances in Booster

- Intrinsic resonances:

$$0+\nu\colon\ |G\gamma| = 4.54 \text{ with } \varepsilon = 0.11 \qquad K = (\gamma-1)mc^2 = 239 \text{ MeV}$$

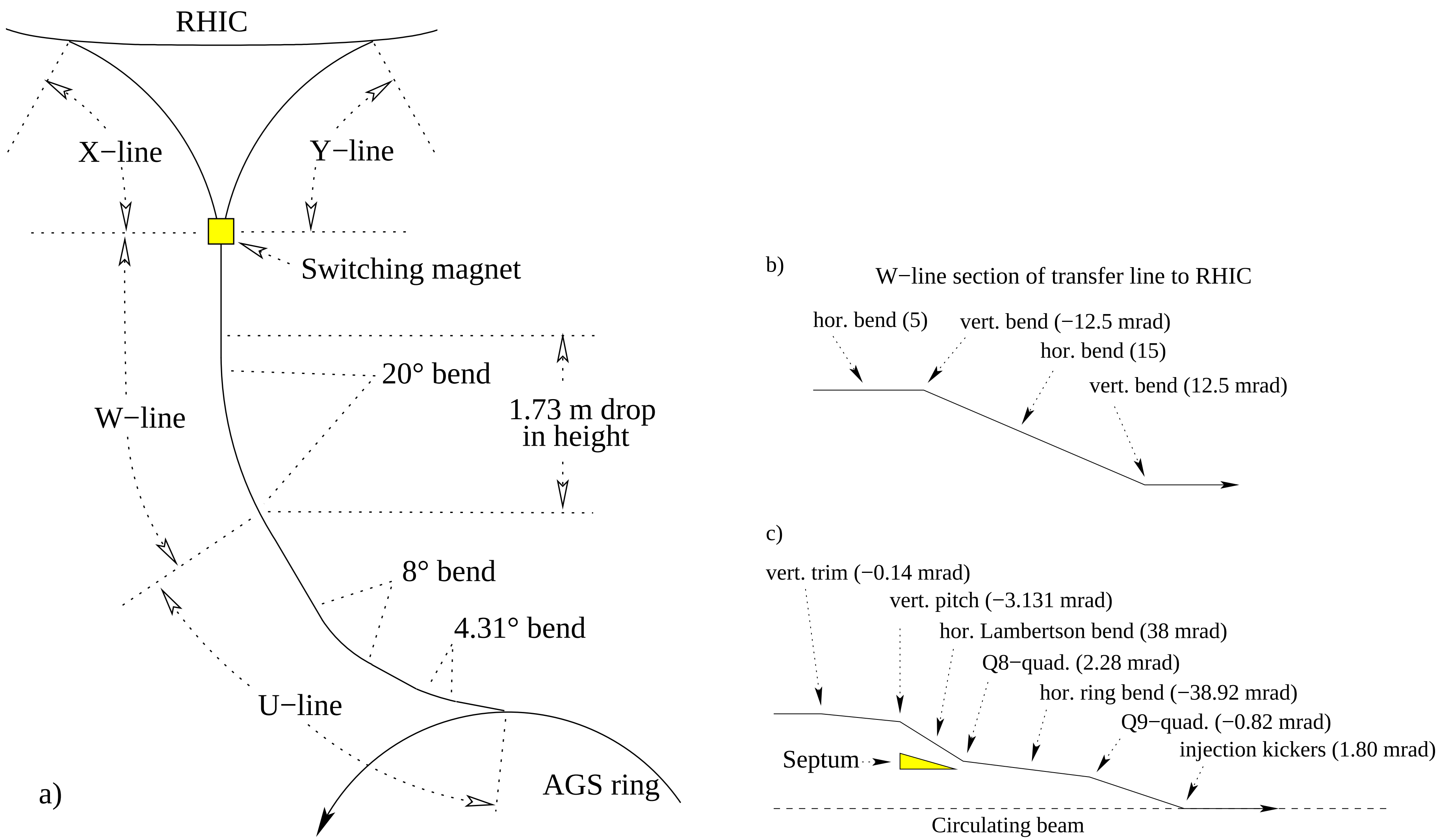
$$12-\nu\colon\ |G\gamma| = 7.46 \text{ with } \varepsilon = 0.009$$

- Can extract just before $12-\nu$.
- Injection will be at $|G\gamma| = 4.187$ which is below $0+\nu$
- There are also the imperfection resonances at 5, 6, and 7 to contend with.
  - More harmonic corrections.
  - Do we need an ac dipole or lower extraction energy?

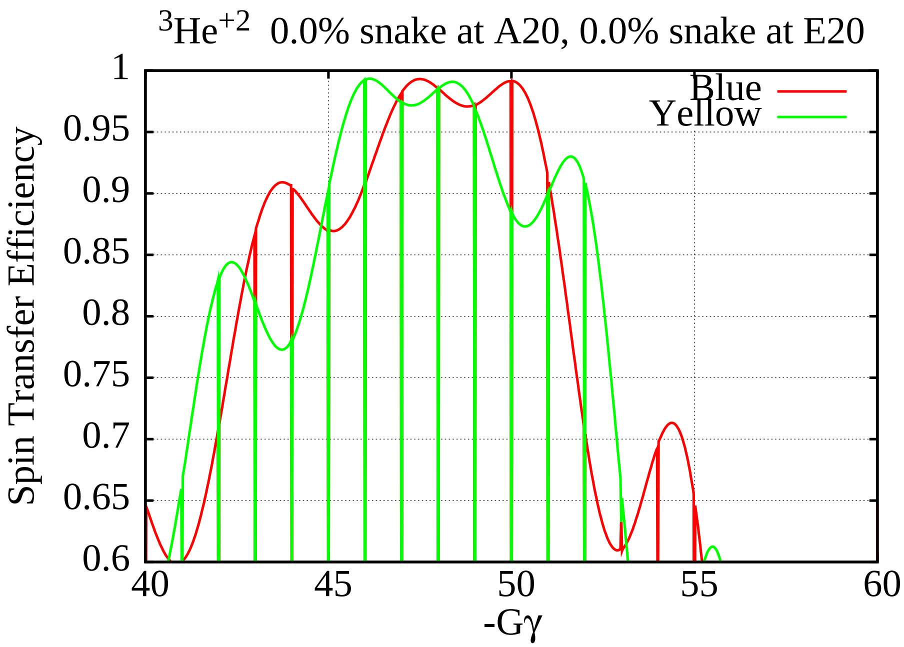
	U/n[GeV]	γ	$G\gamma$	p/q[Tm]
Booster inj	1.030	1.100	-4.6036	2.150
AGS inj	1.678	1.793	-7.5000	6.968
RHIC inj	11.075	11.831	-49.5000	55.216
RHIC max	166.668	178.039	-744.9100	833.904

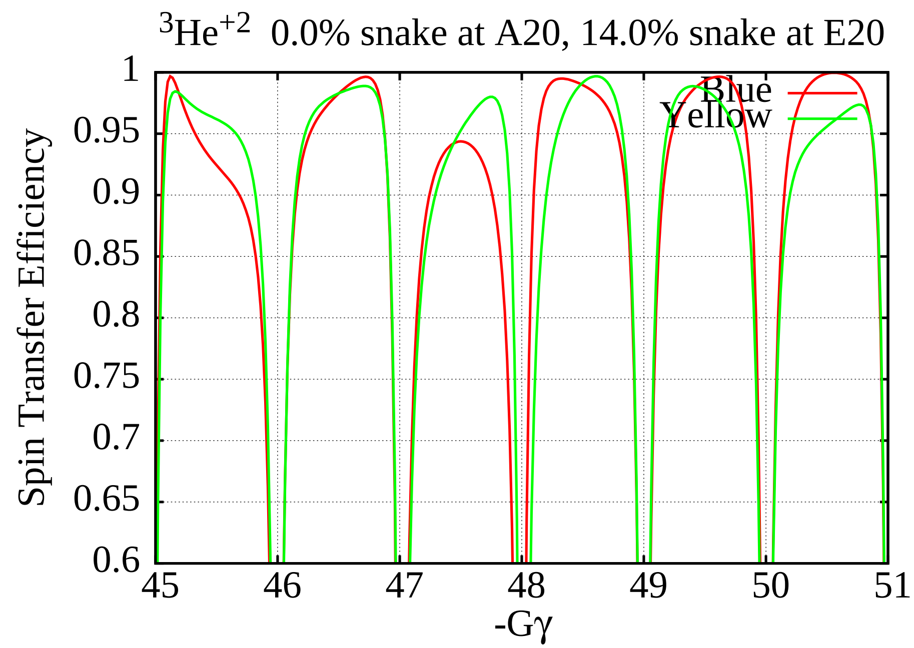
- $^3\mathrm{He}^{+2}$, like heavy ions, injects into RHIC below transition: $\gamma_t \simeq 22.9$.

AGS to RHIC transfer line



Matching Helions from AGS to RHIC





- Without AGS snakes.
- $\frac{p_{inj}}{q} = 55.2$ Tm at -49.5 (proton inj: 79.4 Tm).

- With a 14% snake at E20 (warm)
- $G\gamma = -48.5$ and -49.5 OK.

Need Polarimetry Development for He^3

- Equivalent of p+C CNI polarimeters:
  - Possibility of ^{3}He+C CNI polarimeters.
    T. L. Trueman, "CNI Polarimetry with ^{3}He", `arXiv:0710.3380v1` (2007).
    Can we use the same geometry as for protons?
- Equivalent of H-jet p+p polarimeter for absolute polarization:
  - ^{3}He-jet polarimeter might be feasible.
    Can we have a good calibration of the jet polarization?
- Must have local (relative) polarimeters at STAR and PHENIX.

- Need to have a workshop on ^{3}He polarimetry for RHIC.

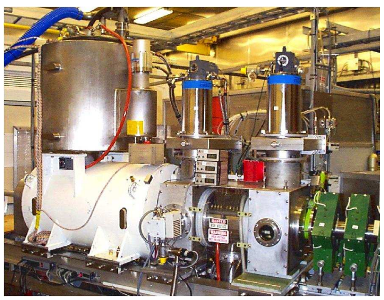
Summary

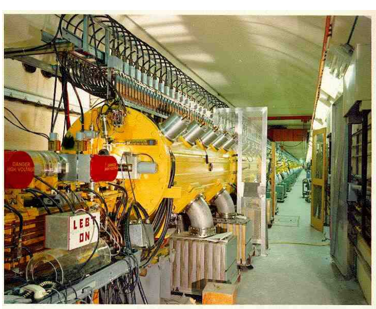
- Deuterons very hard in RHIC — perhaps in a figure-8 ring.

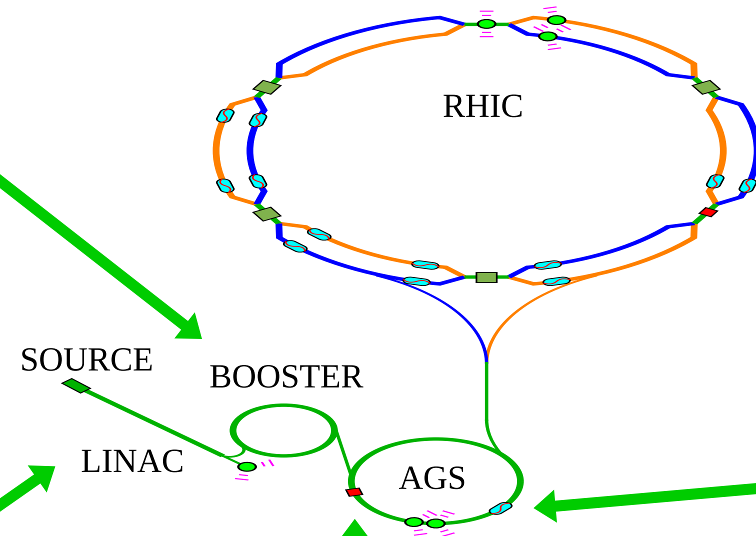
- He^3 looks promising: no real show stoppers.
  - Source: $^3He^{+2}$ OPPIS source — proposal: Milner/Zelenski
    See Anatoli Zelenski's presentation.
  - $|G\gamma|_{max}$ is higher for He^3:
    - More and Stronger resonances in all rings.
  - ◦ ^{3}He polarimeters need to be developed.
  - AGS cold snake may be sufficient at lower field.
    AGS warm snake (fixed field) might be too strong ($\sim 14\%$).
  - AGS injection and extraction spin-matching: not too bad.
    - ◦ Booster to AGS may need matching (depends on AGS snakes).
  - RHIC snakes and rotators will work with lower fields.
  - Lower injection rigidity for RHIC should be OK.
    - Injection orbit excursions reduced.

Backup slides follow

Accelerator Complex (Pol. Protons)







LINAC: Linear Accelerator
AGS: Alternating Gradient Synchrotron
RHIC: Relativistic Heavy Ion Collider

Spin precession equations

- In rest system: $\frac{d\vec{S}^\diamond}{dt^\diamond} = \vec{\mu}^\diamond \times \vec{B}^\diamond$ with $\vec{\mu}^\diamond = \frac{gq}{2m}\vec{S}^\diamond$.
- Thomas-Frenkel-BMT equation and Lorentz force (covariant form):

$$\frac{dS^\mu}{d\tau} = \frac{e}{m}\left[\frac{g}{2}F^{\mu\nu} + \frac{g-2}{2}\left(F^{\mu\nu} + u^\mu\, F^{\nu\kappa}u_\kappa\right)\right]S_\nu$$
$$\frac{du^\mu}{d\tau} = \frac{e}{m}\;\; F^{\mu\nu}u_\nu$$

- T-F-BMT in weird hybrid system (fields in lab; spin at rest):

$$\frac{d\vec{S}^\diamond}{dt} = -\frac{q}{\gamma m}\left[(1+G\gamma)\vec{B}_\perp + (1+G)\vec{B}_\parallel + \left(G\gamma + \frac{\gamma}{\gamma+1}\right)\frac{\vec{E}\times\vec{v}}{c^2}\right]\times\vec{S}^\diamond$$
$$G = \frac{g-2}{2}$$

Warning: Need to be very careful with what $\perp$ and $\parallel$ mean.

Anomalous magnetic moment factor

Calculate g from

$$g = \left(\frac{\mu}{\mu_\mathrm{N}}\right) \times \mu_\mathrm{N} \times \frac{2M}{Ze\hbar I}.$$

$$G = \frac{g-2}{2}$$

- $\mu = g\frac{Ze}{2M}\hbar I$ where I is the nuclear spin quantum number.
- Nuclear magneton: $\mu_\mathrm{N} = e\hbar/2m_p = 31.524512 \times 10^{-9}[\mathrm{eV/T}]$
- Masses, $\hbar$, etc. from NIST 2006 CODATA (http://physics.nist.gov/cuu/Constants/index.html)
- Ratios of magnetic moments to nuclear magneton from N. J. Stone, "Table of nuclear magnetic dipole and electric quadrupole moments", in *Atomic Data and Nuclear Data Tables* **90**, 75 (2005).

SU(2) group generators in rest system

Spin-$\frac{1}{2}$ $(s = \frac{1}{2})$ $\quad \vec{\sigma}$: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (Pauli matrices)

Spin-1 $(s = 1)$ $\quad \vec{\sigma}$: $\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$$\frac{d\psi^\diamond}{d\tau} = \frac{i}{\hbar} \frac{gq}{2m} (\vec{S}^\diamond \cdot \vec{B}^\diamond) \psi^\diamond \qquad \text{with spin operator:} \quad \vec{S}^\diamond = \hbar I \vec{\sigma}$$

$$\psi^\diamond(\tau) = e^{i\kappa(\vec{B}^\diamond \cdot \vec{\sigma})\tau} \psi^\diamond(0) \qquad \text{for a constant } \vec{B}^\diamond$$

$$\kappa = \frac{gq}{2m} I$$

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Higher spins than spin-1/2

T-F-BMT does not cause quantum photon transitions between states.

Protons in RHIC: $\tau_{\mathrm{ST}} = 3 \times 10^{12}$ yr; $\quad \tau_{\mathrm{ST}} \propto \left(\frac{m}{Z}\right)^2 \gamma^{-5}$

Spin-1 spinor: $\psi = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$; T-F-BMT rotates only a and c components.

- $m_s = 0$ component b remains unaffected.

Spin-3/2: $\psi = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

- $m_s = \pm 3/2$ components (a and d) rotate together;
- $m_s = \pm 1/2$ components (b and c) rotate together;
- No radiative transitions between $|m_s| = 3/2$ with $|m_s| = 1/2$.

Formulae for helical dipoles

Parameters for a single RHIC rotator helix [Mike Syphers: SN020]

Pitch: $k = \frac{2\pi}{\lambda}$, $\lambda = 2.41$ m [+(−) for right(left)-handed]

$\kappa = \frac{q}{p}(1 + G\gamma)B$ (simple analytic scaling ignoring longitudinal field)

$\Rightarrow$ $\kappa \sim \frac{q}{p}G\gamma B$ (with more accurate tracking)

Rotation axis: $\hat{r} = \dfrac{k\hat{z} + \kappa\hat{x}}{\sqrt{\kappa^2 + k^2}}$

Precession angle: $\alpha = 2\pi\left(\sqrt{1 + \left(\frac{\kappa}{k}\right)^2} - 1\right)$

Transverse offset: $\Delta x = \dfrac{q}{p}\dfrac{B\ell}{k} = \dfrac{q}{p}\dfrac{\lambda^2}{2\pi}B$

Scaling Snakes to He^3

Scaling of the field at maximum energy:

The maximum rigidity of the beams must the same: $r_{\rm max} = \frac{p}{q} = 834$ Tm

$$(\beta\gamma)_{\rm x} \simeq \frac{Z}{A}(\beta\gamma)_{\rm p}$$

Want the same precession, so κ must be the same.

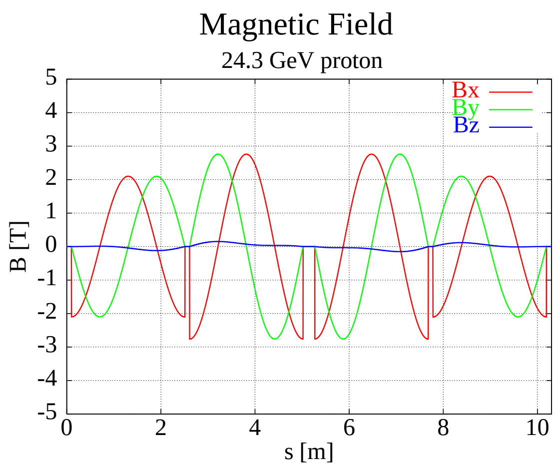
$$B_{\rm x} \simeq \frac{G_{\rm p}\gamma_{\rm p}}{G_{\rm x}\gamma_{\rm x}} B_{\rm p}$$

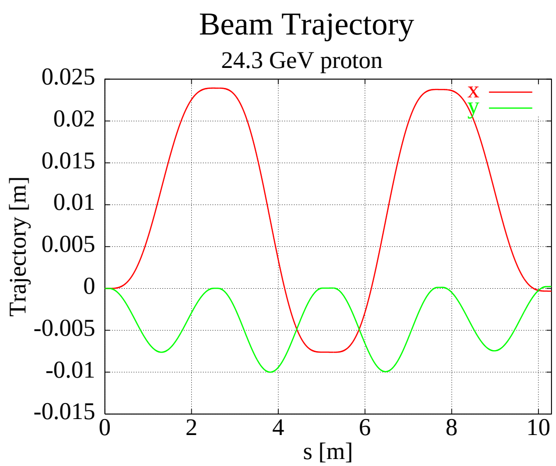
$$B_{\rm He^3} \simeq \frac{AG_{\rm p}}{ZG_{\rm He^3}} B_{\rm p} \simeq -0.643 B_{\rm p}$$

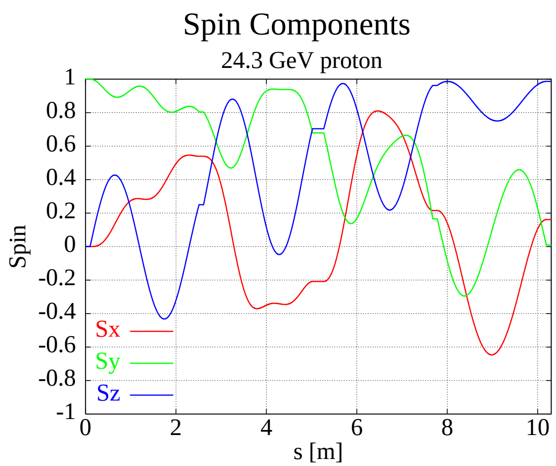
Snake excursion at injection $r_{\rm inj} = 81.1$ Tm (for protons):

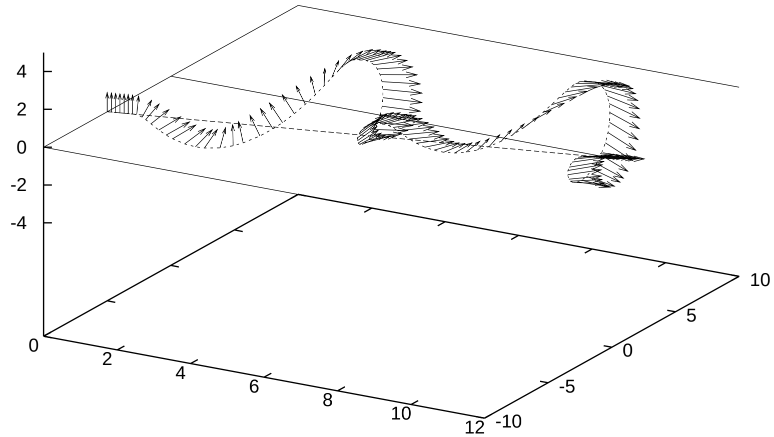
$$\Delta y = \begin{cases} 3.2 \text{ cm}, & \text{for protons} \\ -2.1 \text{ cm}, & \text{for He}^3 \end{cases}$$

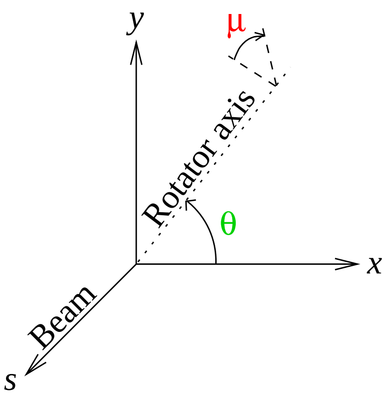
Helical Spin Rotators



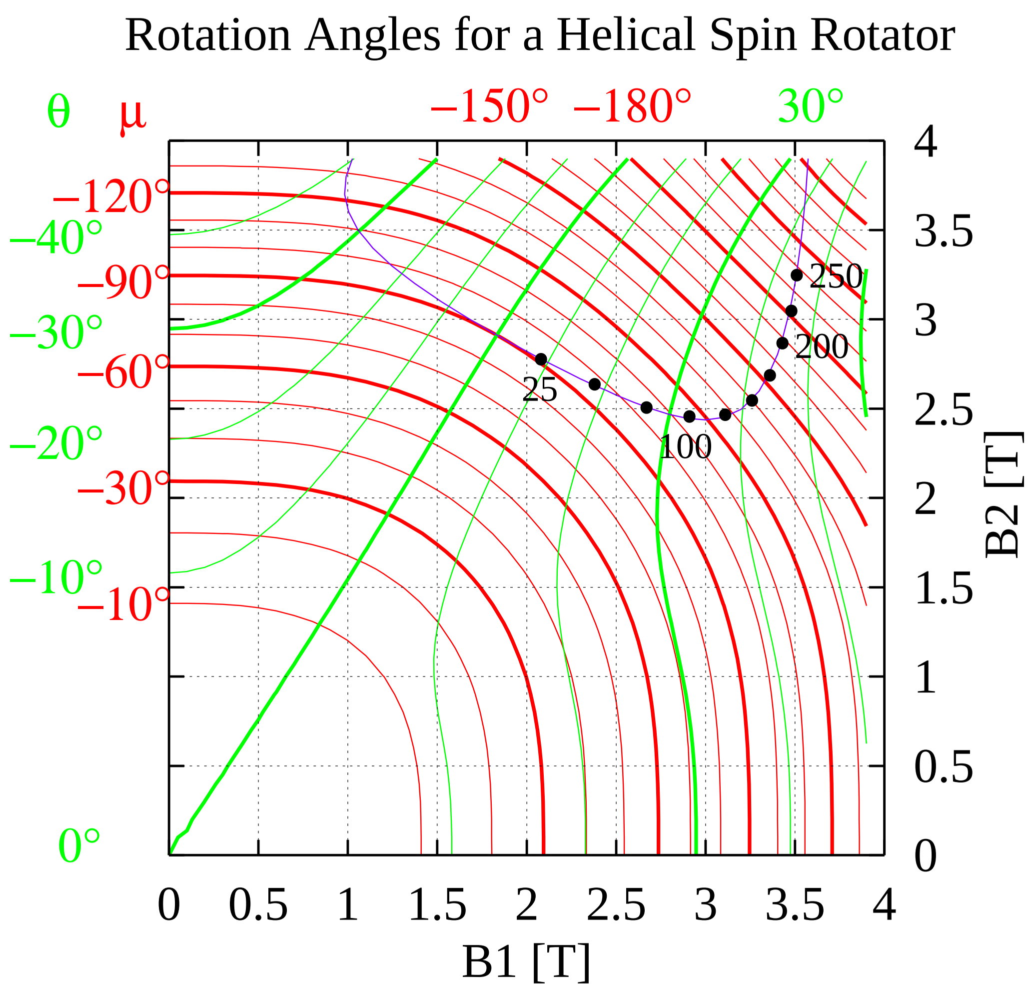








The rotation axis of the spin rotator is in the x-y plane at an angle θ from the vertical. The spin is rotated by the angle μ around the rotation axis.



Note: Purple contour for rotation into horizontal plane. Black dots show settings for RHIC energies in increments of 25 GeV from 25 to 250 GeV.

Rotator Axes and Precession

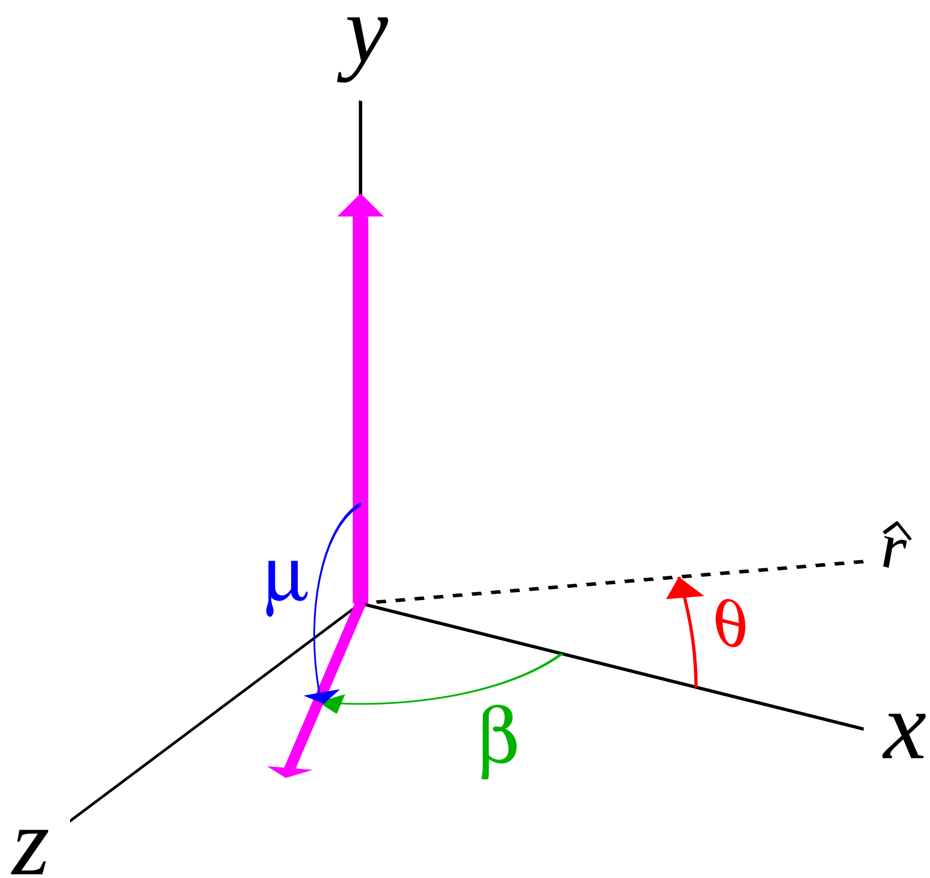
To precess the spin from vertical into the horizontal plane:

$$\sin\beta = \sin\mu\cos\theta$$
$$\cos\mu = -\tan^2\theta$$

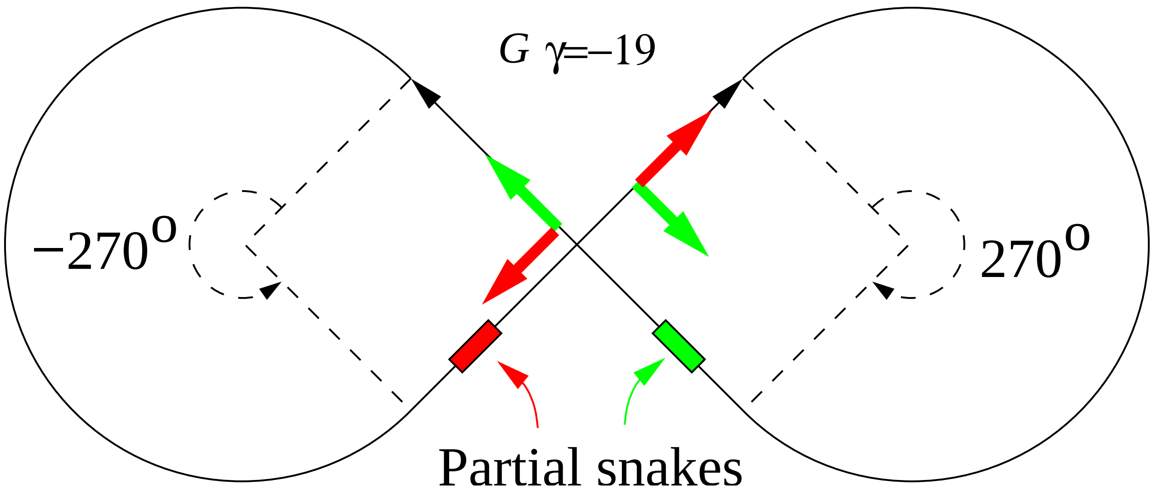
$$\mu \in [90^\circ, 270^\circ]$$
$$\theta \in [-45^\circ, 45^\circ] \cup [135^\circ, 225^\circ]$$

For longitudinal polarization want:

$$\beta = G\gamma \times \theta_{\mathrm{D0DX}}$$
$$\theta_{\mathrm{D0DX}} = 3.675 \text{ mr}$$



Deuterons in a Figure 8



- JLAB (Derbenev)
- With no snakes $\nu_{\mathrm{sp}} = 0$ $\mathbf{R}_{\hat{n}}(\text{full turn}) = \mathbf{I}$.
- Weak snake locks spin.

- At $G\gamma = -19$, the net precession around one arc is $-14\frac{1}{4}$ rotations.
- Switch between red and green snakes to rotate polarization by 90°.
- Any injector ring should also be a figure 8 with partial snakes.